Mathematical Modeling and Performance Analysis of P2P Streaming Networks[1]

Yuliya Gaidamaka, Andrey Samuylov, and Konstantin Samouylov

Peoples' Friendship University of Russia, Telecommunication Systems Department
Ordzhonikidze str. 3
115419 Moscow, Russia
ygaidamaka@mail.ru, asam1988@gmail.com, ksam@sci.pfu.edu.ru

Abstract: In this paper, we present two approaches to the P2P live streaming networks mathematical modeling and performance measures analysis. The first model is described in terms of queuing network with each node corresponding to a TV channel and with users, wandering among the nodes of the network. Given the popularity of channels we obtained normal type approximation of the universal streaming probability for a network with infinite number of users. The second model describes in terms of discrete Markov chain the data exchange process between users in P2P live streaming network with buffering mechanism. While considering user churn in the model we take into account playback lags as well as how user departure affects buffer occupancy and playback.

Keywords: P2P network, live streaming, buffer occupancy, universal streaming playback continuity, Markov chain model, transfer delays, playback lags.

1 Introduction

P2P-network consists of users who make their resources (computing power, memory, network bandwidth) available to other users without the need for central coordination. In P2P-networks, users not only download, but also distribute the downloaded data to other users, thus reducing the load on the server. In P2P-networks, a file is divided into several small blocks of data, known as chunks, and each user downloads the missing file chunks from other users who have already downloaded them.

There are two types of P2P-networks: file-sharing and streaming P2P networks [1,2]. In file-sharing networks, users have to download the entire file before they begin to use it, so a user is not restricted by time to obtain any chunk. In streaming networks, users simultaneously download and play the video stream, so a limit for download time of a chunk is crucial, since every chunk has its playback deadline.

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In file-sharing P2P networks, a file will not be played until at least one chunk is missing. The order a user downloads chunks is not important, and the downloaded chunks are stored on a disk. In streaming P2P networks, each user has a buffer for caching the most recently downloaded data chunks, moreover only the chunks that are yet to be played will be downloaded. In both cases in order to select which chunk to download next, a download strategy, such as Rarest First (RF), Latest First (LF) or Greedy (Gr), is applied [3].

One of the main performance metrics of file-sharing networks is how long it takes to download the whole file (the file download time). In streaming networks, the main performance measures are the startup delay, playback continuity and the probability of universal streaming.

P2P networks performance measures are usually analyzed via using different mathematical models. The so-called fluid models are used to analyze file sharing networks [4-9]. Streaming P2P networks are more strict in respect to performance measures, and their distinctive feature is that they are generally analyzed in discreet time [10-13].

In this paper, we present mathematical models for analyzing the data exchange process in P2P streaming network: the TV channel playback model and the model of peer exchange while playing a video stream.

In section 2, we describe the TV channel playback model that allows to analyze one of the main performance measures – universal streaming. This model is based on a model introduced in [14]. In [15] it was modified in order to take into account the behavior of a single user while switching channels. In section 3, based on [16, 17] we introduce the model of filling the buffer while playing a video stream, which accounts for data transfer delays and the fact that users may leave the network or return. Note that the model described in [16] is developed in terms of a discrete Markov chain that allows us to give a rigorous mathematical description of the download strategy, and to obtain analytical formulas for the main performance measures of the network.

2 Queuing Network Model for Analyzing Universal Streaming in Multi-Channel IPTV Network

In [14] the universal streaming is defined as a probability that all peers receive the selected TV channel at a rate not less than the streaming rate of the channel. In [15] the exact formulas for calculating this probability are derived in case of a finite number of users, as well as its approximation by the normal law in case of a large-scale network.

First we construct a model of a single user behavior in P2PTV-network in the form of closed exponential queuing network, with the set of M $\left(|\mathsf{M}| = M\right)$ nodes, each one corresponding to a TV channel, with a single customer, wandering among the nodes of the network. The model describes the switching process of channels by a single user. Let ρ_m be the relative popularity of m-channel and $y_m \in \{0,1\}$ be the state of the user watching m-channel, $y_m = 1$ if a user is watching m-channel and $y_m = 0$ otherwise and then $\mathbf{y} = (y_m)_{m\in\mathsf{M}}$ is the state vector of the system. Gordon-

Newell theorem for the case of N=1 (one user in the system) implies the following proposition.

Proposition 1: For the queuing network model, describing the behavior of single user, the stationary probability distribution of the system has the product form $P(\mathbf{y}) = \prod_{m\in \mathsf{M}} \rho_m^{y_m}$, $\mathbf{y} \in \{\mathbf{y} : y_m \in \{0,1\},\ \sum_{m\in \mathsf{M}} y_m = 1\}$.

Consider a network with finite number $N < \infty$ of users and denote by $x_{nm} \in \{0,1\}$ the state of the *n*-user on *m*-channel, i.e. $x_{nm} = 1$, if the *n*-user is on *m*-channel, and $x_{nm} = 0$ otherwise. Then the state of the system can be described by the matrix $\mathbf{X} = (x_{nm})_{n\in \mathsf{N},\, m\in \mathsf{M}}$, and the state space of the system will has the following form $\mathsf{X} = \left\{ \mathbf{X} : x_{nm} \in \{0,1\},\ \sum_{m\in \mathsf{M}} x_{nm} = 1,\ n \in \mathsf{N} \right\}$.

Proposition 2: The probability of the state $\mathbf{X}$ of closed network with $N < \infty$ users is given by $P(\mathbf{X}) = \prod_{n\in \mathsf{N}} \prod_{m\in \mathsf{M}} \rho_m^{x_{nm}}$, $\mathbf{X} \in \mathsf{X} = \left\{ \mathbf{X} : x_{nm} \in \{0,1\},\ \sum_{m\in \mathsf{M}} x_{nm} = 1,\ n = 1,...,N \right\}$.

We consider a network with two types of users. Let N^h and N^l be the number of users with high and low upload rates correspondingly. Then $N = N^h + N^l$ is the total number of users in the network. We call the network closed if $N < \infty$, and open otherwise. Parameter $K = N^h / N^l$ stands for the ratio of the number of users with high upload rate to the number of users with low upload rate in a closed network. Let $0 \le \xi_m^h \le N^h$ ($0 \le \xi_m^l \le N^l$) be a random variable of the number of users watching *m*-channel with high (low) upload rate in a closed network. In [15] it is shown that the marginal distribution of the number of users in a network is given by:

$$P_m\left(x_m^i\right) = P\left\{\xi_m^i = x_m^i\right\} = \binom{N^i}{x_m^i} \rho_m^{x_m^i}\left(N^i\right)\left(1 - \rho_m\left(N^i\right)\right)^{N^i - x_m^i}. \tag{1}$$

Then the probability of universal streaming in *m*-channel is defined as:

$$\pi_m = P\left(A_m\right) = P\left\{\left(\xi_m^h, \xi_m^l\right) \in A_m\right\} = \sum_{x_m^h=0}^{N^h} \sum_{x_m^l=0}^{N^l} \mathrm{I}\left(A_m\right) P_m\left(x_m^h\right) P_m\left(x_m^l\right),\ m \in \mathsf{M},$$

$$0 \le x_m^i \le N^i,\quad i \in \{h, l\}\quad , \tag{2}$$

where $\mathrm{I}\left(A_m\right)$ is an indicator function, and

$A_m = \left\{\left(\xi_m^h, \xi_m^l\right):\ 0 \le \xi_m^h \le N^h,\ \ 0 \le \xi_m^l \le N^l,\ \ s_m + \xi_m^h u^h + \xi_m^l u^l \ge \left(\xi_m^h + \xi_m^l\right) R_m\right\}$

is the set of universal streaming states.

Let us consider an open network ($N = \infty$), where $\hat{\xi}_m^h \ge 0$ ($\hat{\xi}_m^l \ge 0$) is a random variable of the number of users with high (low) upload rate watching *m*-channel.

Marginal distribution of the number of users in an open network is derived by passing to the limit the binomial distribution to the Poisson distribution in formula (1):

$$\hat{P}_m\left(x_m^i\right) = P\left\{\hat{\xi}_m^i = x_m^i\right\} = e^{-\gamma_m^i} \frac{\left(\gamma_m^i\right)^{x_m^i}}{x_m^i\,!}, \quad 0 \le x_m^i \le N^i, \quad i \in \{h, l\}, \quad m \in \mathrm{M},$$

where $\gamma_m^i = \lim\limits_{N^i \to \infty} N^i \rho_m\left(N^i\right)$, $i \in \{h, l\}$ is the average number of users watching m-channel.

Assuming that the relation $\gamma_m^h / \gamma_m^l = K$, $m \in \mathrm{M}$ is hold in an open network, we use the Lindeberg-Levy central limit theorem by centering and normalizing the random variables $\hat{\xi}_m^h$ and $\hat{\xi}_m^l$, distributed according to the Poisson law:

$$\hat{Z}_m^i = \frac{\hat{\xi}_m^i - \gamma_m^i}{\sqrt{\gamma_m^i}}, \ i \in \{h, l\}.$$

Obviously, the random variable $\hat{Z}_m^i$ is distributed according to the standard normal law, that is $\hat{Z}_m^i \in N(0,1)$, $i \in \{h, l\}$.

Let $\varepsilon_m = \dfrac{R_m - u^l}{u^h - R_m}$ denotes the ratio between the difference of the streaming rate of m-channel and the low upload rate and the difference of the high upload rate and the streaming rate of m-channel. Let $\delta_m = \dfrac{s_m}{u^h - R_m}$ is the ratio between the server upload rate and the difference of the high upload rate and the streaming rate of m-channel. We define $\hat{Z}_m$ as a linear combination of random variables $\hat{Z}_m^h$ and $\hat{Z}_m^l$: $\hat{Z}_m = \varepsilon_m \hat{Z}_m^l - \sqrt{K}\hat{Z}_m^h$. Note that $\hat{Z}_m$ is distributed according to the normal law, i.e. $\hat{Z}_m \in N\left(0, K + \varepsilon_m^2\right)$, $m \in \mathrm{M}$.

Theorem 1. An open network with users watching m-channel and parameters γ_m^h, γ_m^l, ε_m, and δ_m is in universal streaming state if $\hat{Z}_m \le d_m$, where $d_m = \dfrac{\left(K - \varepsilon_m\right)\gamma_m^l + \delta_m}{\sqrt{\gamma_m^l}}$, $m \in \mathrm{M}$.

Corollary 1. Universal streaming probability in an open network with users watching m-channel is approximated by the normal law $\hat{\pi}_m = \Phi\left(\frac{d_m}{\sqrt{K+\varepsilon_m^2}}\right)$, where $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy$, $m \in \mathrm{M}$.

Next we give an example of calculation of the universal streaming probability π_m for a closed network with M=20 channels and N=1800 users, considering $N^h = N^l = 0,5N$. We assume that, $\gamma_m^h = \gamma_m^l = \rho_m \cdot 0,5N$, so K=1. All the channels have the same streaming rate of $R_m = R = 500$ Kbit/s, $m \in \mathrm{M}$, while user upload rates are $u^l = 100$ Kbit/s and $u^h = 1500$ Kbit/s. The channel popularity follows a Zipf law $\rho_m = \left(m^z \sum_{i=1}^{M} \frac{1}{i^z}\right)^{-1}$, $m \in \mathrm{M}$, with the default Zipf parameter as 1. Figure 1 shows the universal streaming probability π_m calculated according to formulas (1) and (2) and the universal streaming probability $\hat{\pi}_m$ calculated for each channel according to the Corollary 1. The comparison shows that the largest relative error occurs for the least popular 20-channel and does not exceed 1%.

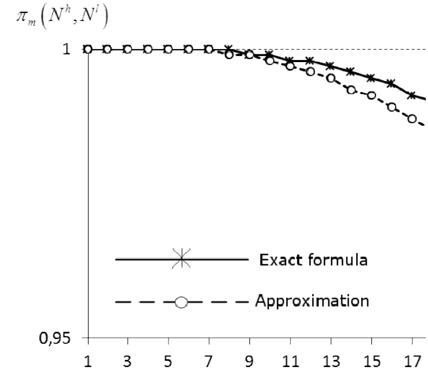

Fig. 1. Universal streaming probability for P2P TV channels

3 Discrete Markov Chain Model for Analyzing Playback Continuity in P2P Live Streaming Network

In this section, we study video data distribution in a P2P streaming network taking into account buffering mechanism. Consider a P2P network with N users present in the network, and a single server, which transmit only one video stream. The process of video stream playback is divided into time slots, the length of each time slot corresponds to the playback time of one chunk. Each user has a buffer designed to accommodate M+1 chunks, where the buffer positions are numbered from 0 to M: 0-position is to store the most recent chunk just received from the server, other m-positions, m=1,…,M-1 are to store chunks, already received during the past time slots or will be downloaded in the coming time slots, and buffer M-position is to store the oldest chunk that will be moved out from the buffer to the player for playback during the next time slot.

Let us specify the actions that the server and users perform during each time slot. At the beginning of each time slot the server randomly selects a user from the network and uploads the newest chunk into his buffer 0-position. Every other user, not chosen by the server during the current time slot, will perform the following actions. If there are empty positions in the user's buffer, i.e. there are missing chunks in his buffer, the user will choose another user (called a target user) from the predefined group of his neighbors randomly in order to download one of the missing chunks from him. If the target user has one of the missing chunks, then the attempt to download from the target user will be successful. If the target user has more than one of the missing chunks, then downloading strategy will define which chunk to download. One of the simplest used strategies is LF strategy. With the LF strategy during any time slot users will try to download the latest useful chunk [18]. A user will not download any chunk in the current time slot at all, if in the current time slot his buffer is filled (there are no empty positions) or if the target user he chose does not have any of the missing chunks. At the end of each time slot, chunks in the buffer of each user will shift one step forward, i.e. chunk in M-position will exit out of the buffer and move to the player for playback, the remaining chunks in other positions will shift one position to the right (towards the end of the buffer) to replace the position freed by its predecessor, and in this case buffer 0-position will be free to accommodate a new chunk from the server at the beginning of the next time slot.

It is assumed that each user may at some time leave the network and stop collaboration with other users, and likewise a user may join the network and begin collaborate with other users. Next we develop a mathematical model for data exchange between users in the form of discrete Markov chain describing the buffer states of all users. Here the model of user behavior, proposed in [15,16], is extended by taking into account data transfer delays (playback lags) that affect the video data exchange process between users as it is shown in fig. 1.

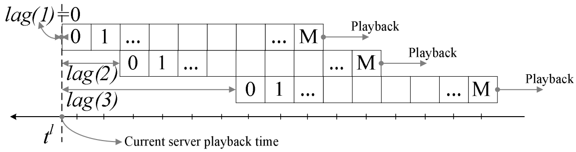

Figure 1. Buffers states mapping with playback lags

For a given network with N users and a single server, vector $\mathbf{z}(n)=(a(n),lag(n),\mathbf{x}(n))$ defines the state of each user (n-user), where $a(n)$ is user on-line indicator ($a(n)=1$ if the user is on-line and $a(n)=0$ otherwise), $lag(n)$ is the data transfer delay from server and $\mathbf{x}(n)=(x_0(n),x_1(n),\cdots,x_{\mathrm{M}}(n))$ is the state of n-user's buffer. Here $x_m(n)$ is the state of n-user's buffer m-position: $x_m(n)=1$, if n-user's buffer m-position is occupied with a chunk, otherwise $x_m(n)=0$. Each user in the network uses buffer positions m=1,…,M to store the downloaded chunks from the network, and uses 0-position only to download a chunk from the server. Note that, if during any time slot M-position is filled, then n-user will watch the video stream without any pause.

Thus the state of the system is defined by $\mathbf{Z}=(\mathbf{a},\mathbf{lag},\mathbf{X})$, where $\mathbf{a}=(a(1),...,a(\mathrm{N}))$ is an indicator vector of users on-line, $\mathbf{lag}=(lag(1),...,lag(N))$ is a vector that defines the delays for each user, and the n-th row of the matrix $\mathbf{X}$ corresponds to the buffer state of n-user, $\dim \mathbf{X}=\mathrm{N}(\mathrm{M}+1)$.

Denote by $M^0(\mathbf{x}(n))$ and $M^1(\mathbf{x}(n))$ the set of all empty and filled positions in n-user's buffer respectively, $M^0(\mathbf{x}(n))=\{m:\ x_m(n)=0,\ m=\overline{1,\mathrm{M}}\}$, $M^1(\mathbf{x}(n))=\{m:\ x_m(n)=1,\ m=\overline{1,\mathrm{M}}\}$, where $M^0(\mathbf{x}(n))\cup M^1(\mathbf{x}(n))=\{1,2,...,\mathrm{M}\}$. Also we denote by $M^{lag(i),lag(j)}$ the set of i-user buffer positions that are available for data exchange with j-user, as due to the data transfer delays not the entire buffer will be available for data exchange. The $M^{lag(i),lag(j)}$ set is shown on fig. 2 and is given by:

$$M^{lag(i),lag(j)}=\begin{cases}\{0,1,\dots,\mathrm{M}-lag(i)+lag(j)\}, & \text{if } lag(i)\ge lag(j)\\ \{lag(i)-lag(j),\dots,\mathrm{M}\}, & \text{if } lag(i)<lag(j)\end{cases} \tag{3}$$

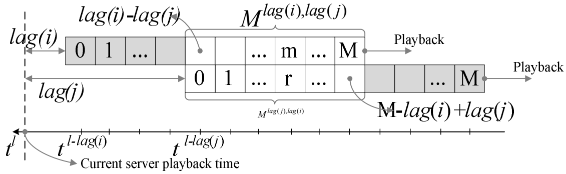

Figure 2. The scheme of the $M^{lag(i),lag(j)}$ set

Then $M^0\left(\mathbf{x}(n)\right)\cap M^{lag(n),lag(h)}$ will be the set of empty positions in n-user's buffer, to which he can download data from h-user, and $M^1\left(\mathbf{x}(h)\right)\cap M^{lag(h),lag(n)}$ will be the set of filled positions in h-user's buffer, that he can upload to n-user. Due to playback lags one and the same data chunk in the buffers of users with different data transfer delays will be located in positions with different indexes. In order to establish a correspondence between these positions we use the following operation: $m = r - lag(n) + lag(h)$. Here m is an index of buffer position for n-user, and r is a corresponding index of buffer position for h-user, $m \in M^{lag(n),lag(h)}$, $r \in M^{lag(h),lag(n)}$. Thereby, the index $m_\delta\left(\mathbf{x}(n),\mathbf{x}(h),lag(n),lag(h)\right)$ of n-user's buffer position to which n-user can download a chunk from h-user is determined by the downloading strategy in use:

$$m_\delta\left(\mathbf{x}(n),\mathbf{x}(h),lag(n),lag(h)\right) = \min\left\{\left(M^0\left(\mathbf{x}(n)\right)\cap M^{lag(n),lag(h)}\right)\cap \right.$$
$$\left.\cap\left\{m : m = r - lag(n) + lag(h), r \in \left(M^1\left(\mathbf{x}(h)\right)\cap M^{lag(h),lag(n)}\right)\right\}\right\}. \quad (4)$$

Denote by $S\mathbf{x}(n)$ the shifting operator of vector $\mathbf{x}(n)$, meaning if $\mathbf{x}(n) = \left(x_0(n), x_1(n), \cdots, x_{M-1}(n), x_M(n)\right)$, then $S\mathbf{x}(n) = \left(0, x_0(n), \cdots, x_{M-1}(n)\right)$. Let t_l be the shifting moment of buffer contents. When constructing the model in a discrete time it is assumed that if at the moment $t_l - 0$ a buffer is in the state $\mathbf{x}(n)$, then at the moment $t_l + 0$ it will be in the state $S\mathbf{x}(n)$.

Let t_l be the shifting moment of buffer contents. When constructing the model in a discrete time it is assumed that if at the moment $t_l - 0$ a buffer is in the state $\mathbf{x}(n)$, then at the moment $t_l + 0$ it will be in the state $S\mathbf{x}(n)$. We assume that a user can leave the network, or join the network only at the moment t_l. Denote by $a^l(n)$ the n-user's on-line indicator at the moment $t_l - 0$. If n-user joined the network at the moment t_l or earlier and never left the network till the moment t_l, then $a^l(n) = 1$, and $a^l(n) = 0$ otherwise. Let $\alpha(n)$ be the probability of n-user joining the network and $\beta(n)$ the probability of n-user leaving the network:

$P\{a^{l+1}(n) = 1 | a^l(n) = 0\} = \alpha(n);$
$P\{a^{l+1}(n) = 0 | a^l(n) = 0\} = 1 - \alpha(n);$

$P\{a^{l+1}(n) = 0|a^l(n) = 1\} = \beta(n)$;
$P\{a^{l+1}(n) = 1|a^l(n) = 1\} = 1 - \beta(n)$.
For simplicity assume that all users join and leave the network with equal probabilities, i.e. $\alpha(n) = \alpha$, $\beta(n) = \beta$, $n = \overline{1, N}$. We also assume that when n-user leaves the network then the corresponding row in matrix $\mathbf{X}$ will be reset i.e. $\mathbf{x}(n) = \mathbf{0}$.

According to the protocol for the distribution of data in P2P live streaming networks with a buffering mechanism, in the interval $[t_l, t_{l+1})$, which corresponds to the l-th time slot, the server and users perform the following actions.

1. At the moment t_l, an off-line user decides to join the network with probability α and an on-line user decides to leave the network with probability β.
2. At the moment t_l, for all users the shift of the buffer content takes place:
   - Chunk in buffer M-position if present will be sent for playback;
   - All other chunks in other buffer positions will be shifted one position to the right, i.e. towards the end of the buffer;
   - Buffer 0-position will be emptied.
3. At the moment $t_l + 0$, server chooses one user randomly and uploads a chunk for the current time slot to his buffer 0-position. If server has chosen n-user, then $x_0(i) = 1$ at the moment $t_{l+1} - 0$.
4. Each user (n-user), not chosen by the server, randomly chooses one of his neighbors (as an h-user), and according to download strategy δ tries to download missing data chunks. If according to the strategy δ index $m_\delta(\mathbf{x}(n), \mathbf{x}(h), lag(n), lag(h))$ in n-user's buffer is chosen, then he will download that chunk from h-user. No actions are performed otherwise.

Denote by $\mathbf{Z}^l = (\mathbf{a}^l, \mathbf{lag}^l, \mathbf{X}^l)$ the network state at the moment $t_l - 0$ and then the set $\{\mathbf{Z}^l\} := \{\mathbf{Z}^l,\ l \geq 0\}$ forms a Markov chain over state space Z with one class $\tilde{Z}$ of essential states, $\tilde{Z} \subset Z$.

Let $\pi^l(\mathbf{Z})$ be the probability that Markov chain $\{\mathbf{Z}^l\}_{l \geq 0}$ during l-th time slot is in state $\mathbf{Z}$, i.e. $\pi^l(\mathbf{Z}) = P\{\mathbf{Z}^l = \mathbf{Z}\}$ and $\Pi^{l,l+1}(\mathbf{Z}, \mathbf{Y})$ be the corresponding transition probability.

Note that the transition probability $\Pi^{l,l+1}(\mathbf{Z}, \mathbf{Y})$ depends on $m_\delta(\mathbf{x}(n), \mathbf{x}(h), lag(n), lag(h))$, i.e. on the strategy in use, on playback lags, and on α and β, i.e. on the user joining and leaving probabilities. The probability distribution $\pi^l(\mathbf{Z})$ satisfies the Kolmogorov-Chapman equations:

$$\pi^{l+1}(\mathbf{Y}) = \sum_{\mathbf{Z} \in Z} \pi^l(\mathbf{Z}) \Pi^{l,l+1}(\mathbf{Z}, \mathbf{Y}),\ \mathbf{Y} \in Z,\ l \geq 0. \qquad (5)$$

4 Playback Continuity Analysis and Some Case Studies

One of the main performance measures of P2P live streaming networks is the probability $V(n)$ of playback continuity, which is the probability that buffer M-

position of n-user is filled with the corresponding chunk for playback at the end of any time slot. To find this probability, we introduce the $h_n^i(\mathbf{Z})$ function, which corresponds to the number of users who have a chunk in their buffer i-position, from which n-user can download in accordance with the downloading strategy δ when the network is in state $\mathbf{Z}$:

$h_n^i(\mathbf{Z}) = \sum_{h=1,\cdots,\mathrm{N}:\, h\neq n,\ a^l(h)=1} \delta_{m_\delta(\mathbf{x}(n),\mathbf{x}(h)),i}$, $\mathbf{Z} \in Z$,

where $\delta_{j,i} = \begin{cases} 1, & j = i, \\ 0, & j \neq i. \end{cases}$

Let $\mathrm{N}(\mathbf{a}^l) = \sum_{j=\overline{1,\mathrm{N}}} a^l(j)$ be the number of on-line users when the network is in state $\mathbf{Z}^l = \left(\mathbf{a}^l, \mathbf{lag}^l, \mathbf{X}^l\right)$. Now we define the probability $Q_n^l(i)$ that during the l-th time slot the chunk which n-user can download to his buffer i-position is available in the network. Due to the dependency of this probability on the downloading strategy δ we can interpret $Q_n^l(i)$ as the probability that n-user will select i-position and successfully download a chunk from the target user during the l-time slot. If $\mathrm{N}(\mathbf{a}^l) \geq 2$, then one can obtain the following formula

$$Q_n^l(0) = 0,\ \ Q_n^l(i) = \sum_{\mathbf{Z}\in Z} \pi^l(\mathbf{Z}) \cdot \frac{h_n^i(\mathbf{Z})}{\mathrm{N}(\mathbf{a}^l) - 1},\ \ i = 1, \cdots, M. \tag{6}$$

Denote by $p_0^l(n,i)$ ($p_1^l(n,i)$) the probability that i-position of n-buffer is empty (filled) during l-th time slot. Then one can obtain a recursive relation for calculating the buffer state probabilities in a following form:

$$\begin{aligned} & p_1^l(n,0) = \frac{1}{\mathrm{N}}, \\ & p_1^{l+1}(n,i+1) = p_1^l(n,i)\cdot(1-\beta) + \\ & \quad + p_0^l(n,i)\cdot(1-\beta)\cdot Q_n^l(i) + \alpha \cdot Q_n^l(i), \\ & i = 0, \cdots, M-1. \end{aligned} \tag{6}$$

Assume that the equilibrium distribution of the Markov chain $\{\mathbf{Z}^l\}$ exists. Denote by $p_1\ (n,i) = \lim_{l\to\infty} p_1^l(n,i)$ the probability that i-position of n-buffer is filled and by $p_0\ (n,i) = \lim_{l\to\infty} p_0^l(n,i)$ the probability that i-position of n-buffer is empty. Then from formula (6) we obtain the following:

$$\begin{aligned} & p_1\ (n,0) = \frac{1}{\mathrm{N}}, \\ & p_1\ (n,i+1) = p_1\ (n,i)\cdot(1-\beta) + \\ & \quad + p_0\ (n,i)\cdot(1-\beta)\cdot Q_n\ (i) + \alpha\cdot Q_n\ (i), \\ & i = 0, \cdots, \mathrm{M}-1. \end{aligned} \tag{7}$$

Let us denote by the probability that n-user is watching video without pauses during playback, i.e. probability of playback continuity, then we have the following formula

(8)

Hence the desired probability measures of the considered model are obtained, and next we provide some case studies for P2P live streaming network performance analysis.

For our case study we chose N=300, M=40, and the number of neighbors is equal to 60. We consider a case where all users are present in the network and do not depart, i.e. and . Also for simplicity the set of all users N is split into three non-overlapping groups, assuming that the delay for all users in one group is the same, i.e. $N = \bigcup_{i=1}^{3} N_i$, $lag(k) = lag(k')$, $k, k' \in N_i$ $i = 1,2,3$. Consider the case where the delay of the first group is zero and the delay of the second and third groups are 10 and 20 time slots respectively. The graphs in fig. 3 and in fig. 4 shows that the group with the largest delay (the third group) will have the greatest probability of watching video stream without pauses in playback. This can be explained by the fact that any data chunk will be highly available among users from the first and second groups by the time it is requested by the users from the third group. These graphs don't lead to the conclusion which of two strategies is better. It is for further study.

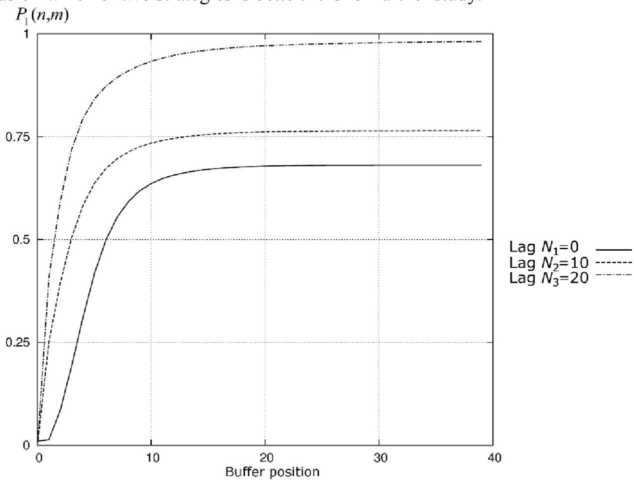

Fig. 3. Buffer occupancy probability in case of LF strategy

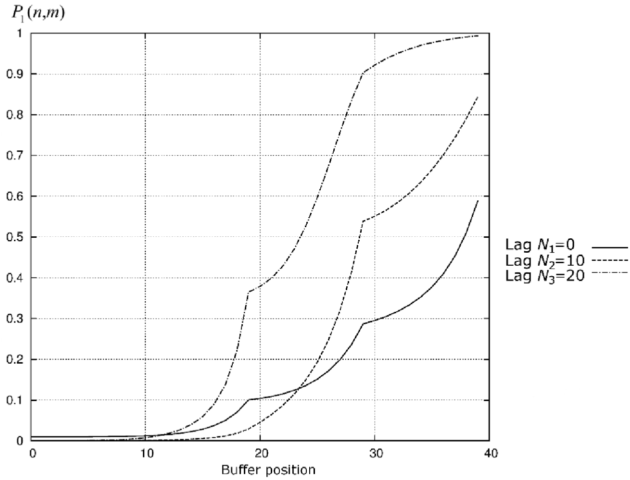

Fig. 4. Buffer occupancy probability in case of Gr strategy

5 Conclusion

In this paper two models of data exchange between users of P2P live streaming network was presented. The first one was developed using queuing network theory and the second one was developed in terms of discrete Markov chain, through which the formulas for the analysis of the system performance measures were obtained. In case when α=0 and β=0, we provide some calculations results for two most popular downloading strategies, i.e. . The direction for our further study is to take into account different downloading rates of users in P2P network and to analyze both performance measures, i.e. universal streaming and playback continuity, within the unique mathematical model.

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